Mixed estimation technique in semi-parametric space-time point processes for earthquake description

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Abstract: An estimation approach for the semi-parametric intensity function of a particular space-time point process is introduced. In particular we want to account for the estimation of parametric and nonparametric components simultaneously, applying a forward predictive likelihood to semi-parametric models. For each event, the probability of being a background event or one belonging to a seismic sequence is therefore estimated.

Keywords: nonparametric estimation; forward predictive likelihood; ETAS model; point process; earthquakes.

1 Introduction

Prediction of large earthquakes is often complicated by the presence of clusters of aftershocks, superimposed to the persistent background seismicity. Indeed, earthquake clusters, formed by the main event of each sequence, its foreshocks and its aftershocks, may complicate the statistical analysis of the background seismic activity that might be related to changes in the tectonic field.

Since the seismogenic features controlling the kind of seismic release of background and clustered seismicity are not similar, sometimes a preliminary subdivision or declustering of a seismic catalog is useful to study separately the features of independent events and triggered ones (Adelfio et al., 2006).

In previous works (Adelfio, 2010; Adelfio et al., 2010) we proposed a clustering technique to separate and find out the two main components of seismicity, i.e. the background seismicity and the triggered one.

Adelfio et al. (2010) presented a seismic sequences detection technique based on MLE of parameters, that identifies the conditional intensity function of a model describing the seismic activity as a clustering-process, like ETAS model (Epidemic Type Aftershocks-Sequences model; Ogata, 1988).

In Adelfio (2010) nonparametric methods are used to estimate the intensity function of a space-time point process and clustering results are interpreted by a second-order diagnostic approach (Adelfio and Schoenberg, 2009). Zhuang et al. (2002) proposed a stochastic method associating to each event a probability to be either a background event or an offspring generated by other events. A probabilistic clustering approach, providing an uncertainty about an object's class membership, can be provided by latent clustering analysis (Fraley and Raftery, 2002).
In this paper, we propose an estimation of the space-time intensity of the generating point process of the different components, that accounts simultaneously for the estimation of parametric and nonparametric components applying a forward predictive likelihood estimation approach to semi-parametric models (Chiodi and Adelfio, 2011). According to this approach we estimate, for each event, the probability of being a background event or one belonging to a seismic sequence.
In section 2 some formal definitions of point processes are recalled. A new method for nonparametric estimation is introduced in section 3; the simultaneous approach for nonparametric and parametric estimation is proposed in section 4.

2 Intensity function in point processes and ETAS model

Point process is a random collection of points, each one representing the time and space coordinates of a single event.
Let $Z^d = S^{d-1} \times T$ be a general $d-$dimensional closed region, with S^{d-1} a two or three dimensional space. Any analytic space-time point process is uniquely characterized by its associated *conditional intensity function* (Daley and Vere-Jones, 2003) defined as the frequency with which events are expected to occur around a particular location in time and space, conditional on the prior history $\mathcal{H}_t$ of the point process up to time t, i.e.:

$$\lambda(\mathbf{z}) = \lambda(\mathbf{s}, t|H_t) = \lim_{\Delta t, \Delta \mathbf{s} \to 0} \frac{\mathrm{E}\left[\#(t+\Delta t, \mathbf{s}+\Delta \mathbf{s}|H_t)\right]}{\Delta t \Delta \mathbf{s}}$$

where H_t is the space-time occurrence history of the process up to time t, $\Delta t, \Delta \mathbf{s}$ are time and space increments, $\mathrm{E}\left[\#(t+\Delta t, \mathbf{s}+\Delta \mathbf{s}|H_t)\right]$ is the history-dependent expected number of events occurring in the volume $\{[t, t+\Delta t) \times [\mathbf{s}, \mathbf{s}+\Delta \mathbf{s}]\}$. Generally, intensities $\lambda(\mathbf{z})$ depend on some unknown parameter $\boldsymbol{\psi}$, so that we have $\lambda(\mathbf{z}, \boldsymbol{\psi})$. For example, in a semi-parametric context, $\boldsymbol{\psi}$ could contains smoothing parameters.
Let denote a generic estimator of $\boldsymbol{\psi}$, based on observations until t_k, by $\hat{\boldsymbol{\psi}}(H_{t_k}) \equiv \hat{\boldsymbol{\psi}}((\mathbf{z}_1, \mathbf{z}_2, ..., \mathbf{z}_i, ..., \mathbf{z}_k)$.
Assume that a realization of the process is observed in the space region $\Omega_{\mathbf{s}}$ and the time interval $(T_0; T_{max})$. The log-Likelihood for the point pro-

cess, given the k observed values $\mathbf{z}_i$ and computed using the estimator $\hat{\boldsymbol{\psi}}(\mathbf{z}_1, \mathbf{z}_2, ..., \mathbf{z}_i, ..., \mathbf{z}_k)$ is:

$$\log L(\hat{\boldsymbol{\psi}}(H_{t_k}); H_{t_k}) = \sum_{i=1}^{k} \log \lambda(\mathbf{z}_i; \hat{\boldsymbol{\psi}}(H_{t_k})) - \int_{T_0}^{T_{max}} \int_{\Omega_\mathbf{s}} \lambda(\mathbf{z}; \hat{\boldsymbol{\psi}}(H_{t_k}))\, d\mathbf{s}\, dt \tag{1}$$

In seismological context, the Epidemic Type Aftershocks-Sequences (ETAS) model is widely used (Ogata, 1988). The conditional intensity function of the ETAS model is defined as the sum of a term describing the spontaneous activity (background) and one relative to the induced seismicity:

$$\lambda_{\boldsymbol{\theta}}(x, y, t, m|\mathcal{H}_t) = \mu f(x, y) + \tau_{\boldsymbol{\phi}}(t, x, y) \tag{2}$$

with $\boldsymbol{\theta} = (\boldsymbol{\phi}, \mu)^{\mathrm{T}}$, that is the vector of parameters of the induced intensity ($\boldsymbol{\phi}$) together with the parameter of the background general intensity (μ) and

$$\tau_{\boldsymbol{\phi}}(t, x, y) = \sum_{t_j < t} g(t - t_j; \boldsymbol{\phi}) s(x - x_j, y - y_j|m, \boldsymbol{\phi}).$$

In the ETAS model, background seismicity is assumed to be stationary in time, while the occurrence rate of aftershocks at time t, following the earthquake of time t_j and magnitude m_j, is described by the following parametric model:

$$g(t - t_j|m_j) = \frac{\kappa\ e^{(\alpha-\gamma)\ (m_j - m_0)}}{(t - t_j + c)^p}, \quad \text{with} \quad t > t_j$$

where p is useful for characterizing the pattern of seismicity, indicating the decay rate of aftershocks in time.

For the spatial distribution, conditioned to magnitude of the generating event, the following distribution is often used:

$$s(x - x_j, y - y_j|m_j) = \left\{ \frac{(x - x_j)^2 + (y - y_j)^2}{e^{\gamma\ (m_j - m_0)}} + d \right\}^{-q}$$

It relates the occurrence rate of aftershocks to the mainshock magnitude m_j, through the parameters α, γ. m_0 is a given lower threshold of magnitude, d and q two parameters related to the spatial influence of the mainshock.

The simultaneous estimation of the background intensity and triggered intensity components of ETAS model is a crucial statistical issue. While the first component $f(x, y)$ is usually estimated by nonparametric techniques, $\boldsymbol{\theta}$ is estimated by ML approach. Zhuang et al. (2002) estimated the probability for each event of being a background event ($\rho_i, i = 1, ..., n$) in order to provide a random classification of events and obtain a thinned catalog, that includes events with a bigger probability of being mainshock, which spatial intensity is described by nonhomogeneous Poisson process.

In this paper, according to Console et al. (2010), we use ρ_i as weights for the kernel estimation of the background seismicity to get a simultaneous estimate of the intensity components of the ETAS model (2). For nonparametric estimation we propose the use of an estimation procedure based on the subsequent increments of likelihood obtained adding an observation one at a time, reported in the next section.

3 Forward predictive likelihood (FLP)

To estimate the nonparametric component we use an approach proposed in Chiodi and Adelfio (2011) to measure the ability of the observations and estimation until t_k to give information on the next observation.
Let $\hat{\boldsymbol{\psi}}(H_{t_k})$ be smoothing constants in a nonparametric context, based on the observed history up to t_k.
Let $\log L(\hat{\boldsymbol{\psi}}(H_{t_k}); H_{t_{k+1}})$ be the likelihood computed on the first $k+1$ observations, but using the estimates based on first k. We measure the *predictive information* of the first k observations on the $k+1$-th as:

$$\delta_{k,k+1}(\hat{\boldsymbol{\psi}}(H_{t_k}); H_{t_{k+1}}) = \log L(\hat{\boldsymbol{\psi}}(H_{t_k}); H_{t_{k+1}}) - \log L(\hat{\boldsymbol{\psi}}(H_{t_k}); H_{t_k}),$$

This leads to a technique similar to cross-validation, but applied only on future observations. Therefore, we choose $\widetilde{\boldsymbol{\psi}}(H_{t_k})$ which maximizes:

$$FLP_{k_1,k_2}(\hat{\boldsymbol{\psi}}) \equiv \sum_{k=k_1}^{k_2} \delta_{k,k+1}, \tag{3}$$

where $k_1 = \left[\frac{n}{2}\right]$ and $k_2 = n-1$.
In previous applications (Adelfio and Chiodi, 2011), on the basis of the measure in (3), we observed that the bandwidths estimated by FLP approach produced better kernel estimates (in terms of MISE) of space-time intensity functions than classical methods.

4 Alternating estimation of components

In order to estimate the different components of the ETAS model (2), we here propose to alternate the standard parametric likelihood method (to estimate the parameters of the offsprings component) with the FLP approach (to estimate the background intensity).
Given a catalog of n seismic events and set $v = 1$, let $f^{(1)}(x, y)$ be a starting estimation of the background seismicity, obtained by kernel estimators. The $v - th$ iteration of the simultaneous estimation of nonparametric and parametric components proceeds as follows:

1. Get the ML estimator $\hat{\boldsymbol{\theta}}^{(v)}$ of the parameters of the ETAS model, numerically maximizing the likelihood (1).

2. Estimate $\rho_i^{(v)} = \frac{\mu f^{(v)}(x_i,y_i)}{\lambda_{\hat{\boldsymbol{\theta}}^{(v)}}(x_i,y_i,t_i,m_i|\mathcal{H}_t)}$, $i = 1, ..., n$, for each point of the catalog, on the basis of the estimated parameters. $\rho_i^{(v)}$ is used as a vector of weights for the nonparametric estimation of the background seismicity.

3. Update the estimation of the background seismicity $f^{(v+1)}(x, y)$, through weighted kernel estimator with weights $\rho_i^{(v)}$.

    - Compute the estimated triggered intensity $\tau_{\hat{\boldsymbol{\phi}}^{(v)}}(t_i, x_i, y_i)$ for each point of the catalog.
    - Estimate an optimal smoothing vector $\psi^{(v)}$ of the kernel estimator, maximizing the (3) and holding $\tau_{\hat{\boldsymbol{\phi}}^{(v)}}(t_i, x_i, y_i)$ fixed for the whole iteration.

4. Update v and start a new iteration, until some convergence rule is reached. Convergence is judged comparing the values of ETAS components in consecutive iterations, checking also the increase in the overall likelihood function.

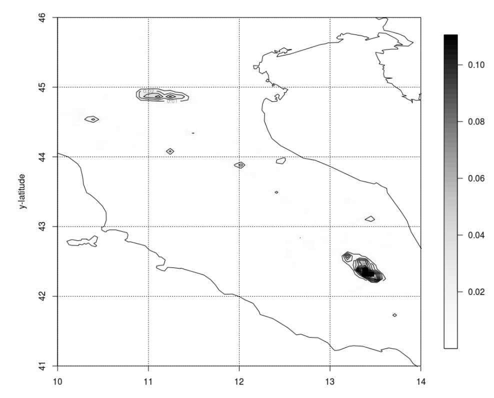

FIGURE 1. Estimated triggered intensity of Italian seismicity (2005-2012).

As an example of application we apply the proposed approach to the catalog of Italian seismic events recorded from 2005 to 2012. The estimate of the only triggered intensity function for a restricted area, reported in figure 1, shows high picks of intensity in correspondence of focal areas of the Italian seismicity, i.e. L'Aquila and Reggio Emilia, where two big sequences of events occurred in 2009 and 2012, respectively. The estimated model seems to follow adequately the seismic activity of the observed area, characterized by highly variable changes both in space and in time. Indeed, because of its flexibility, the estimation approach provides a good fitting to local space-time changes, to analyze possible correlation between the estimated intensity function and particular distributions of some structural features (i.e. geological structures) of the studied region.

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