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A statistical physics view of financial fluctuations: Evidence for scaling and universality

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Abstract

The unique scaling behavior of financial time series have attracted the research interest of physicists. Variables such as stock returns, share volume, and number of trades have been found to display distributions that are consistent with a power-law tail. We present an overview of recent research joining practitioners of economic theory and statistical physics to try to understand better some puzzles regarding economic fluctuations. One of these puzzles is how to describe outliers, i.e. phenomena that lie outside of patterns of statistical regularity. We review recent research, which suggests that such outliers may not in fact exist and that the same laws seem to govern outliers as well as day-to-day fluctuations.

Keywords: Econophysics; Power-laws; Fat tails; Critical dynamics

1. Introduction

The statistical properties of the time series of financial variables are quite unique. Stock returns, for example, display tails that are much more pronounced than a simple Gaussian; indeed, events such as the 1987 stock market crash where the daily move of the leading S&P 500 index recorded a magnitude of about 20 standard deviations signifies the non-Gaussian nature of these fluctuations. An analysis of correlations in stock price fluctuations shows an even more subtle picture. Although stock returns are only short-range correlated, their magnitudes (e.g. absolute values) display a long-range correlation which remains significant even after several months [1–5].

Consider the S&P 500 stock index as a function of time and the simple uncorrelated biased random walk, and plot not the absolute value of the index but instead the *change* in the index (i.e. the numerical derivative, the "return"). We normalize that by the standard deviation. We look over a 13-year period (Fig. 1) and see, e.g., that on Black Monday the fluctuations were more than 30 standard deviations (both positive and negative) for the day, and we also see a very noisy signal. The striking thing is to look at the other curve, the uncorrelated random walk, and see the Gaussian distribution for the fluctuations—which rarely display fluctuations greater than five standard deviations. The

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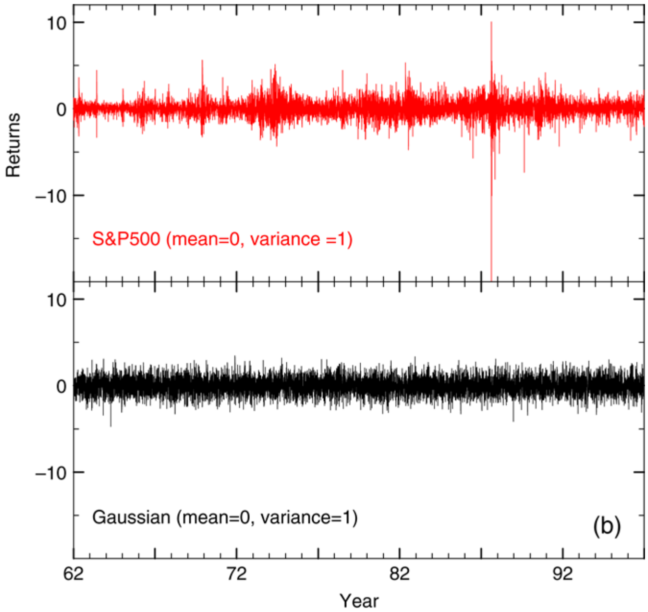

Fig. 1. The S&P 500 index is the sum of the market capitalizations of 500 companies. Shown is the sequence of 10 min returns for the S&P 500, normalized to unit variance, compared with sequence of i.i.d. Gaussian random variables with unit variance, which was proposed by Bachelier as a model for stock returns. The large fluctuations seen in 1987 are due to the market crash of October 19. Note that, in contrast to the top curve, there are no "extreme" events predicted by the bottom curve.

"outliers" that the economists are content to live with are any fluctuations of the actual data that are greater than five standard deviations. In this 13-year period, there are exactly 64, i.e., 2^6. If we count only those fluctuations of the actual data that are greater than ten standard deviations, we get exactly 8, i.e., 2^3. If we count only those that are greater than 20, we get 1, i.e., 2^0: Black Monday. Each time we double the x-axis we change the y-axis by a power of 2^3. At the beginning of this paper, we made a reference to a power law of the form $f(x) = x^{-\alpha}$ with $\alpha = 3$, which corresponds to a functional equation of the form $f(\lambda x) = \lambda^{-3} f(x)$. The possibility that economic data might obey such a scaling was pointed out in 1963 by Mandelbrot in his study of cotton price fluctuations [6]. If we replace our visual examination of these two graphs with a close computer analysis of not just the the S&P 500 stock index, but also of every stock transaction over an extended time period (approximately 10 GB of data), we find [7–10] that the actual graph depicting the number of times a fluctuation exceeds a given amount as a function of that amount is perfectly straight on log–log paper *out to* 100 *standard deviations* (Fig. 2). The slope of the line, α, is indistinguishable from the value $\alpha = 3$ that we deduced from visual inspection. Note that this slope is significantly larger (by almost a factor of two) than the slope found by Mandelbrot in his research on cotton prices. Note also that our slope is outside the Lévy stable regime [11].

This is how we find laws in statistical physics, but finding them is only the first part – the empirical part – of our task. The second part – the theoretical part – is understanding them. Accordingly, we have been seeking to develop a theoretical framework within which to interpret these new empirical facts, and recently some progress is beginning to occur [12–16].

When we studied critical phenomena, the empirical part was a very important contributor towards our ultimate understanding of phase transitions and critical phenomena. Since markets are complex systems with many interacting agents, it is perhaps not unreasonable to examine economic phenomena within the conceptual framework of scaling and universality [1,3,4,6,17–24]. The amassing of empirical facts led to the recognition of regularities to which certain approaches could be applied, e.g., the Widom scaling hypothesis and the Wilson renormalization group. So also in economics, we can perhaps first discover empirical regularities – e.g., the inverse cubic law – that will prove useful in ultimately understanding the economy. We wish wecould say that we already have an explanation for this inverse cubic law, but we can't. We have the beginnings of an explanation [13,12], but it is only the beginning since the current theory explains the inverse cubic law of price changes, as well as the "half cubic law" of trade volume but does not explain

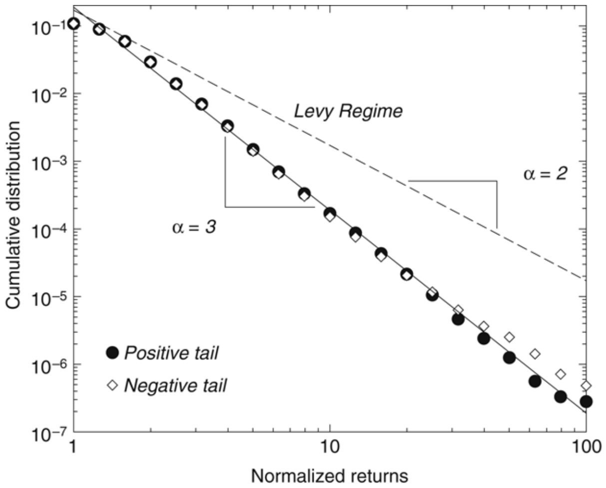

Fig. 2. Cumulative distributions of the positive and negative tails of the normalized returns of the 1000 largest companies in the TAQ database for the 2-year period 1994–1995. The solid line is a power-law regression fit in the region $2 \leq x \leq 80$.

the strange nature of the temporal correlations. The autocorrelation function of price changes decays exponentially in time so rapidly that after 20 min it is at the level of "noise." However the autocorrelation function of changes in the *absolute value* of the price (called the "volatility") decays with a power law with exponent approximately 0.3 [25].

Similarly, our group's initial research in economics – also guided by Pareto's principles – has largely been concerned with establishing which systems display scaling phenomena, and with measuring the numerical values of the exponents with sufficient accuracy that one can begin to identify universality classes if they exist. Economic systems differ from the often-studied physical systems in that the number of subunits are considerably smaller—in contrast to macroscopic samples in physical systems that contain a huge number of interacting subunits, as many as Avogadro's number 6×10^{23}. In contrast, in an economic system, one initial work was limited to analyzing time series comprising an order of magnitude of 10^3 terms, and even nowadays with high frequency data being the standard, one may have maybe 10^8 terms. Scaling laws of the form of (1) are found that hold over a range of a factor of $\approx 10^6$ on the x-axis [45,75,93,91,92]. Moreover, these scaling laws appear to be universal in that they, like the Pareto scaling law, hold for different countries [26], for cities [27–32], other social organizations [33–37], and even for bird populations [38].

Recent attempts to make models that reproduce the empirical scaling relationships suggest that significant progress on understanding firm growth may be well underway [39–44], leading to the hope of ultimately developing a clear and coherent "theory of the firm." One utility of the recent empirical work is that now any acceptable theory must respect the fact that power laws hold over typically six orders of magnitude; as Axtell put the matter rather graphically: *"the power law distribution is an unambiguous target that any empirically accurate theory of the firm must hit"* [45].

With this background on power laws and scale invariance in geometry and in economics, we turn now to the well-studied problem of finance fluctuations, where a consistent set of empirical facts is beginning to emerge. One fact that has been confirmed by numerous, and mostly independent, studies is that stock price fluctuations are characterized by a scale-invariant cumulative distribution function of the power law form (1) with $\alpha \approx 3$ [7,8,46]. This result is also universal, in the sense that this inverse cubic law exponent is within the error bars of the results for different segments of the economy, different time periods, and different countries—and is the same for stock averages as different as the S&P and the Hang Seng [10].

Newcomers to the field of scale invariance often ask why a power law does not extend "forever" as it would for a mathematical power law of the form $f(x) = x^{-\alpha}$. This legitimate concern is put to rest by by reflecting on the fact that power laws for natural phenomena are not equalities, but rather are asymptotic relations of the form $f(x) \sim x^{-\alpha}$. Here, the tilde denotes *asymptotic equality*. Thus $f(x)$ is not "approximately equal to" a power law, so the notation $f(x) \approx x^{-\alpha}$ is erroneous. Similarly, $f(x)$ is not proportional to a power law, so the notation $f(x) \propto x^{-\alpha}$ is also erroneous. Rather, asymptotic equality means that $f(x)$ becomes increasingly like a power law as $x \to \infty$. Moreover, crossovers abound in financial data, such as the crossover from power law behavior to simple Gaussian behavior as the time horizon Δt over which fluctuations are calculated increases beyond about a year (i.e., the power law behavior holds for time horizons up to a month or even a year, but for horizons exceeding a year there is a distinct crossover to Guassian behavior. Such crossovers are characteristic also of other scale-free phenomena in the physical sciences [17, 18], where the Yule distribution often proves quite useful.

For reasons of this sort, standard statistical fits to data are erroneous, and often give distinctly erroneous values of the exponent α. Rather, one reliable way of estimating the exponent α is to form successive slopes of pairs of points on a log–log plot, since these successive slopes will be monotonic and will converge to the true asymptotic exponent α. One finds that successive slopes for the empirical data converge rapidly to a value $\alpha \approx 3$ while successive slopes for the model diverge. While it is clear that a simple three-factor model [47] cannot generate power law behavior, it is less clear why the empirical data analyzed appear at first glance to be well approximated by the model. The first fact is that the region of linearity of the data is not so large as in typical modern studies—because the total quantity of data analyzed is not that large, since only a low-frequency time series comprising daily data is used. Only 28,094 records are analyzed [47] (not 4×10^7 as in recent studies [8,10]) and the model simulations are presented for limited sample size. The second fact is that when one superposes a curved line (the model) on a straight line (the data), the untrained eye is easily tempted to find an agreement where none exists—and closer inspection of Figs. 2–5 of Ref. [47] reveals actually a rather poor agreement between model and data due to the pronounced downward curvature of the model's predictions [48].

2. Universality of the distributions of returns and number of trades

To test the universality of the exponent ζ_R, recent work [49] analyzes detailed trade-by-trade data from three *distinct* markets: (i) 1000 major USA stocks for the 2-year period 1994–1995 ($\approx 10^8$ records), (ii) 85 major stocks traded on the London Stock Exchange for the 2-year period 2001–2002 which form part of the FTSE 100 index ($\approx 4 \times 10^7$ records), and (iii) 13 major stocks traded on the Paris Bourse that form part of the CAC 40 index for the 4.7-year period 3 Jan 1995–22 Oct 1999 ($\approx 2 \times 10^7$ records). To examine the behavior of the distributions over a larger time horizon, we analyze daily data from the CRSP database for 422 stocks for the 35-year period Jan 1962–Dec 1996.

We find that both the negative as well as the positive tails of the distributions of returns display power-law tails, with mutually consistent values of $\zeta_R \approx 3$ for all three markets. We perform similar analyses of the related micro-structural variable, the number of trades $N \equiv N_{\Delta t}$ over time interval Δt, and find a power-law tail for the cumulative distribution $P\{N > x\} \sim x^{-\zeta_N}$, with values of ζ_N that are consistent across all three markets analyzed. Our analysis of US stocks shows that the exponent values ζ_R and ζ_N do not display systematic variations with market capitalization or industry sector. Moreover, since ζ_R and ζ_N are remarkably similar for all three markets, our results support the possibility that the exponents ζ_R and ζ_N are universal (Figs. 3 and 4).

3. Power-law distribution of volume and its universality

While the power-law tails of the distribution of stock returns $P\{R > x\} \sim x^{-\zeta_R}$ are becoming increasingly well documented, less understood are the statistics of other closely-related micro-structural variables such as q_i, the number of shares exchanged in trade i (termed the *trade size*) and $Q_{\Delta t}(t) = \sum_{i=1}^{N} q_i$, the total number of shares exchanged as a result of the $N = N_{\Delta t}$ trades occurring in a time interval Δt (termed *share volume*).

Previous analysis [50] for USA stocks reports power-law distributions

$$P\{q > x\} \sim x^{-\zeta_q} \tag{1}$$

for trade size q and

$$P\{Q > x\} \sim x^{-\zeta_Q} \tag{2}$$

for share volume Q with average tail exponents $\zeta_q = 1.53 \pm 0.07$ and $\zeta_Q = 1.7 \pm 0.1$ both belonging to the Lévy stable domain [0, 2].

Recent work [51] analyzes the statistical properties of trade size $q \equiv q_i$ and share volume $Q \equiv Q_{\Delta t}(t)$ by analyzing trade-by-trade data from three large databases representing three *distinct* markets: (i) 1000 major USA stocks for the 2-year period 1994–95; (ii) 85 major UK stocks for the 2-year period 2001–02; and (iii) 13 major Paris Bourse stocks for the 4.5-year period 1994–99.

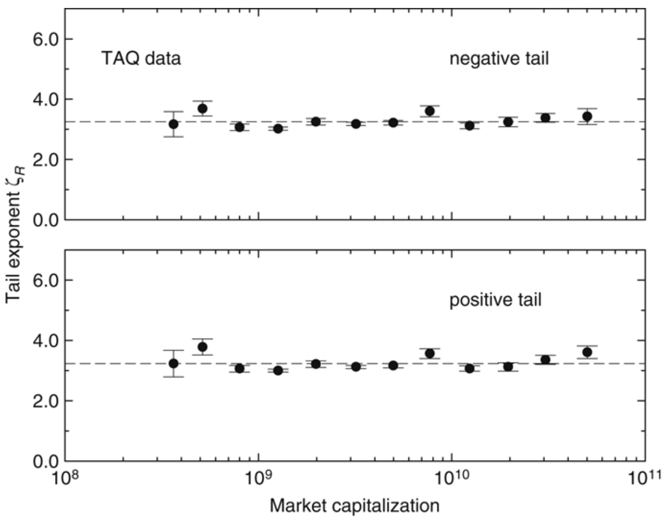

Fig. 3. Estimates of exponent ζ_R that describe the tail behaviors of the $\Delta t = 15$ min returns for 1000 largest US stocks from the TAQ database shows no clear dependence on market capitalization. The bottom panel shows the positive tail exponents, and the top panel the negative tail. Each point shows the average value of ζ_R for each market capitalization group, and the groups are spaced uniformly on a logarithmic scale. We find no systematic dependence. To confirm, we perform a regression $A + B \log x$ without any binning (not shown) and find no significant dependence: we obtain an estimate of $B = -0.04 \pm 0.04$ (positive tail) and -0.01 ± 0.04 (negative tail) with negligible values of R^2.

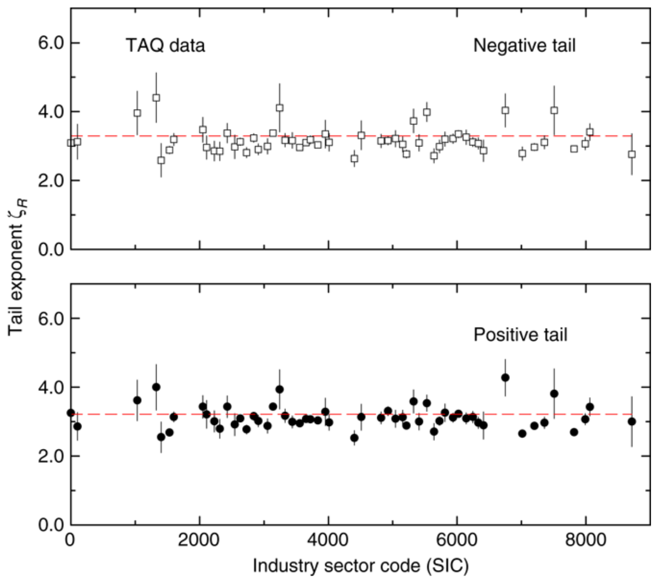

Fig. 4. The tail exponent as a function of the Standard Industry Code (SIC) shows no clear dependence on industry sector. Here we have binned using the first 2 digits of the SIC code, which shows major industry sectors. Deviations from the mean are statistical; points farthest from the mean have large standard errors and occur when only a few stocks contribute. The points at SIC code 0 show the 73 stocks in our sample of 1000 for which we did not have the corresponding SIC codes.

Our analysis of the exponent estimates of ζ_q suggests that the exponent value is *universal* in the following respects: (a) ζ_q is consistent across stocks within each of the three markets analyzed, and also across different markets (b) ζ_q does not display any systematic dependence on market capitalization or industry sector (Fig. 5). We next analyze the distributions of share volume $Q_{\Delta t}$ over fixed time intervals and find that for all three markets, $P\{Q > x\} \sim x^{-\zeta_Q}$ with exponent $\zeta_Q < 2$ within the Lévy stable domain. To test the validity for $\Delta t = 1$ day of the power-law distributions found from tick-by-tick data, we analyze a fourth large database, the CRSP database, containing daily USA data, and confirm a value for exponent ζ_Q within the Lévy stable domain (Fig. 6).

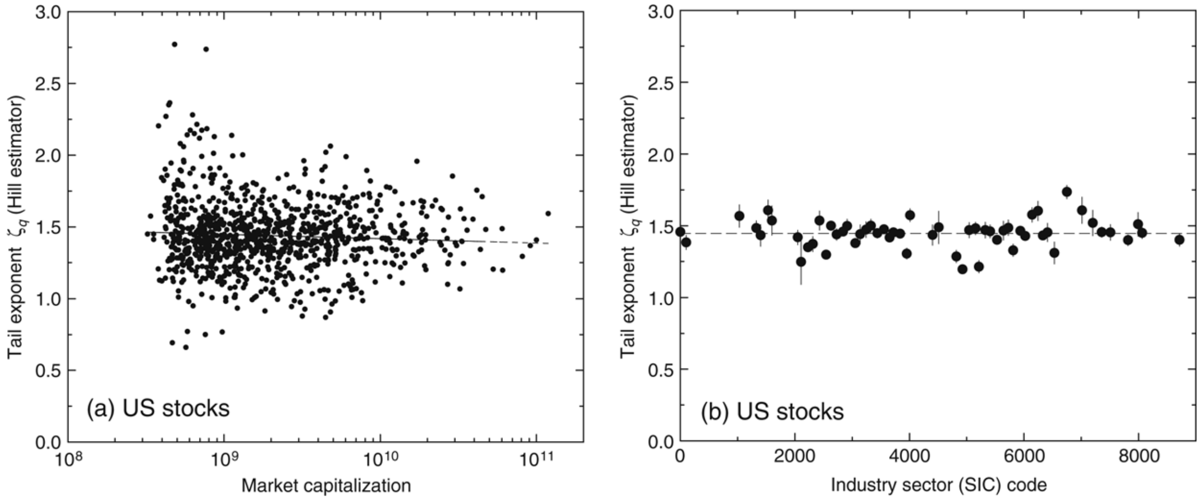

Fig. 5. (a) Hill estimate of exponents ζ_q for the 1000 USA stocks plotted against the average market capitalization for each stock. The line shows a logarithmic regression which shows no significant dependence (we obtain a slope -0.013 ± 0.006 with $R^2 = 0.004$). Note that the exponents in this plot are obtained using Hill's estimator using a tail threshold 5 times the average value. Using estimation thresholds of up to 10, we obtain an average value in the range 1.45–1.67 with no significant dependence on the capitalization. (b) Exponents ζ_q as functions of the SIC code show no clear dependence on the industry sector. Here we have binned using the first 2 digits of the SIC code which shows major industry sectors. The points at SIC code 0 show the 73 stocks in our sample of 1000 for which we did not have the corresponding SIC codes.

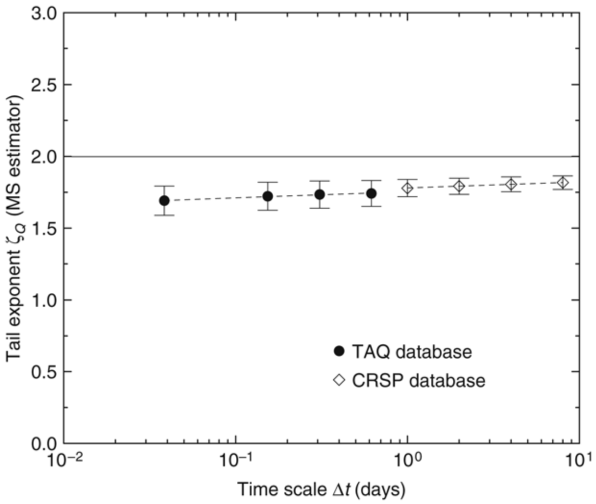

Fig. 6. Meerschaert–Scheffler (MS) estimate of the power-law exponent ζ_Q for time scales Δt ranging from 15 min up to 8 days shows stable behavior within the Lévy stable domain. Here we have used the TAQ database for $\Delta t < 1$ day and the CRSP database for $\Delta t > 1$ day. The exponents seem mutually consistent within error-bars for both the TAQ data and the CRSP data. The dashed lines shows a regression $y = A \log x + B$, and we obtain $A = 0.018 \pm 0.002$ for $\Delta t < 1$ day and $A = 0.019 \pm 0.001$ for $\Delta t > 1$ day.

4. Quantifying fluctuations in market liquidity: Analysis of the bid-ask spread

The primary function of a market is to provide a venue where buyers and sellers can transact. The more the buyers and sellers at any time, the more efficient the market is in matching buyers and sellers, so a desirable feature of a competitive market is liquidity, i.e., the ability to transact quickly with small price impact. To this end, most exchanges have market makers (eg., "specialists" in the NYSE) who provide liquidity by selling or buying according to the prevalent market demand. The market maker sells at the "ask" (offer) price A and buys at a lower "bid" price B; the difference $s \equiv A - B$ is the bid-ask spread.

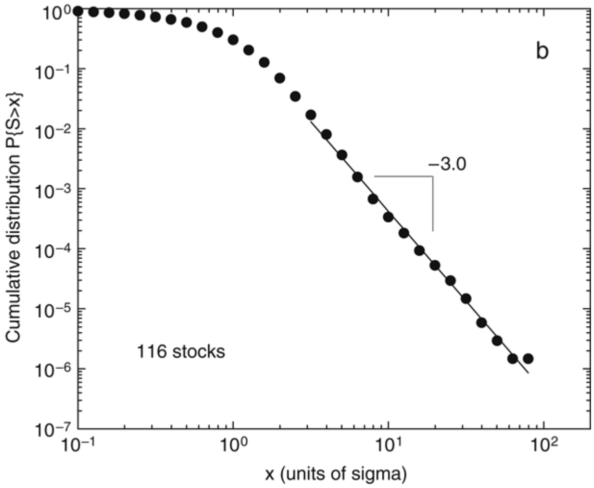

Fig. 7. Log–log plot of the cumulative distribution of the bid-ask spread $S_{\Delta t}$, which is normalized to have a zero mean and unit variance, for all 116 stocks in our sample for the 2-year period 1994–1995. A power law fit in the region $x > 3$ gives a value for the exponent $\zeta_S = 3.0 \pm 0.1$. Fits to individual distributions give similar results for the exponent values.

The ability to buy at a low price and sell at a high price is the main compensation to market makers for the risk they incur while providing liquidity. Therefore, the bid-ask spread must cover costs incurred by the market maker [52–58] such as: (i) order processing costs, e.g., costs incurred in setting up, fixed exchange fees, etc.; (ii) the risk of holding inventory, which is related to the volatility; and (iii) adverse information costs, i.e, the risk of trading with a counter-party with superior information. Since the first component is a fixed cost, the interesting dynamics of liquidity are reflected in (ii) and (iii). Analyzing the statistical features of the bid-ask spread thus also provides a way to understand information flow in the market.

The prevalent bid-ask spread reflects the underlying liquidity for a particular stock. Quantifying the fluctuations of the bid-ask spread thus offers a way of understanding the dynamics of market liquidity. Using quote data for the 116 most-frequently traded stocks on the New York Stock Exchange over the 2-year period 1994–1995, Plerou et al. [59] have recently analyzed the fluctuations of the average bid-ask spread S over a time interval Δt. They find that S is characterized by a distribution that decays as a power law $P\{S > x\} \sim x^{-\zeta_S}$, with an exponent $\zeta_S \approx 3$ for all 116 stocks analyzed (Fig. 7). Their analysis of the autocorrelation function of S shows long-range power-law correlations, $\langle S(t)S(t+\tau)\rangle \sim \tau^{-\mu}$, similar to those previously found for the volatility. They also examine the relationship between the bid-ask spread and the volume Q, and find that $S \sim \log Q$. They find that a similar logarithmic relationship holds between the transaction-level bid-ask spread and the trade size. They also show that the bid-ask spread and the volatility are also related logarithmically. Finally, they study the relationship between S and other indicators of market liquidity such as the frequency of trades N and the frequency of quote updates U, and find $S \sim \log N$ and $S \sim \log U$.

5. Quantifying demand

One reason why the economy is of interest to statistical physicists is that, like an Ising model, the economy is a system made up of many subunits. The subunits in an Ising model are called spins, and the subunits in the economy are buyers and sellers. During any unit of time, these subunits of the economy may be either positive or negative as regards perceived market opportunities. People interact with each other, and this fact often produces what economists call "the herd effect". The orientation of whether we buy or sell is influenced not only by our neighbors but also by news. If we hear bad news, we may be tempted to sell. So the state of any subunit is a function of the states of all the other subunits and of a field parameter.

On a qualitative level, economists often describe a price change as a hyperbolic-tangent-like function of the demand. The catch is that "demand" is not quantified. So one of the first things we had to do was quantify demand [60].

We did this by analyzing huge databases comprising every stock bought or sold—which gives not only the selling price and buying price, but also the asking price and the offer price. If we go to the open market to buy presents, we will often be given an asking price we are not willing to pay, and we may counter with a much smaller offer.

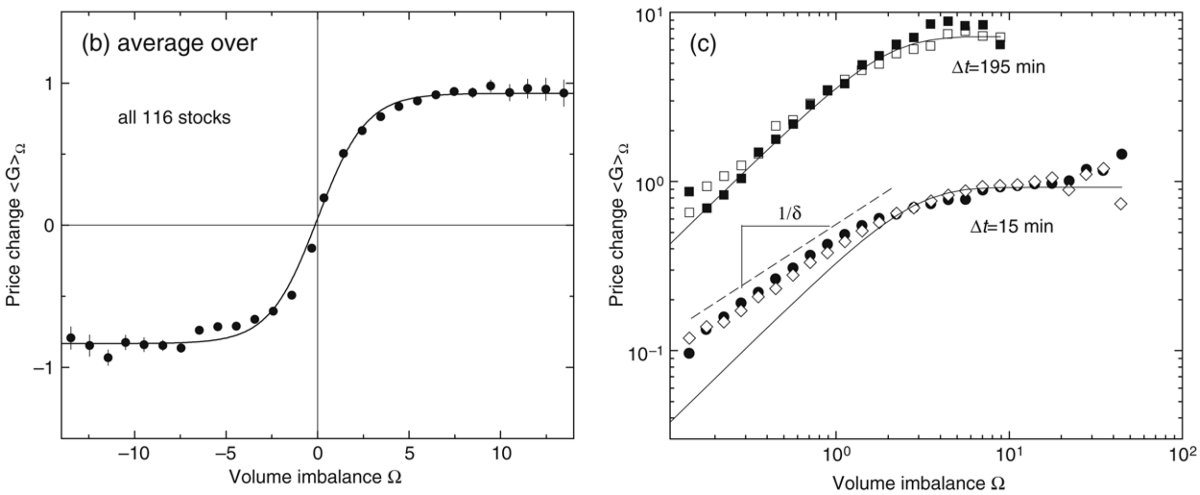

Fig. 8. (a) Conditional expectation $\langle G\rangle_{\Omega}$ averaged over all 116 stocks studied, over a time interval $\Delta t = 15$ min, where Ω is defined as the difference in number of shares traded in buyer and seller initiated trades. We normalize G to have a zero mean and unit variance. Since Ω has a tail exponent $\zeta = 3/2$ which implies divergent variance, we normalize Ω by the first moment $\langle|\Omega - \langle\Omega\rangle|\rangle$. We calculate G and Ω for $\Delta t = 15$ min. The solid line shows a fit to the function $B_0 \tanh(B_1 \Omega)$. (b) $\langle G\rangle_{\Omega}$ on a log–log plot for different Δt. For small Ω, $\langle G\rangle_{\Omega} \simeq \Omega^{1/\delta}$. For $\Delta t = 15$ min, find a mean value $1/\delta = 0.66\pm0.02$ by fitting $\langle G\rangle_{\Omega}$ for all 116 stocks individually. The same procedure yields $1/\delta = 0.34\pm0.03$ at $\Delta t = 5$ min (interestingly close to the value of the analogous critical exponent in mean field theory). The solid curve shows a fit to the function $B_0 \tanh(B_1\, \Omega)$. For small Ω, $B_0 \tanh(B_1\, \Omega) \sim \Omega$, and therefore disagrees with $\langle G\rangle_{\Omega}$, whereas for large Ω the fit shows good agreement. For $\Delta t = 195$ min($\frac{1}{2}$ day) (squares), the hyperbolic tangent function shows good agreement.

Ultimately when the sale is struck, the price may be above the midpoint between the asking price and the offer – and we assign a variable $a_i = +1$ to the sale – if below the midpoint, $a_i = -1$. If we sum all these indices a_i over a time interval Δt

$$H \equiv \sum_{i-1}^{N} a_i = \begin{cases} + & \text{[Big demand]} \\ - & \text{[Small demand]} \end{cases}$$
$$N = N_{\Delta t} = \text{Number of sales in } \Delta t,$$

then we can calculate the analog of a magnetic field, which provides a way of quantifying demand. If most of the a_i are positive, the field will be positive, and vice versa. A hint that this definition using magnetic fields makes sense is the fact that a plot of price change as a function of the "magnetic field" variable defined above remarkably resembles a plot of the magnetization of a magnet as a function of the magnetic field [60] (Fig. 8). The full implications of the remarkable observation that a plot of price change as a function of the "magnetic field" resembles a plot of the magnetization in a magnet are a challenging problem.

6. Cross-correlations among fluctuations of different stocks

A useful software package could determine how the fluctuations of one stock price correlate with those of another. This question of cross-correlation is one we have been studying [8,9,61–67]. To quantify cross-correlations, we draw a circle corresponding to the stock price x, and draw a second circle corresponding to the stock price x, say, five minutes later. If we make the difference in the radii proportional to G, the stock price change, then we can think of the market as thousands of circles, each growing and shrinking—a kind of pulsation that is a function of time. The key is that these correlations change in time. Car sales by Ford and GM may be anti-correlated during some time periods and positively correlated during others.

The standard approach to this problem is to calculate, by brute force, a huge square matrix that has as many rows as there are companies in the database. Each element of the matrix is the correlation between the price change of company i and the price change of company j, but to find a genuine correlation we have to be able to distinguish between correlations from coincidences. In order to do that we draw on something developed by Wigner in his work in nuclear physics—random matrix theory. Random matrix theory compares the matrix calculated by brute force from stock market data with a random matrix that also has 1000 rows and 1000 columns—but with every number generated randomly. Somewhere hidden in the huge matrix calculated by brute force from stock market data are the true correlations. To uncover them, we first diagonalize the matrix in order to determine its eigenvalues, and then make a histogram that gives the number of times each given eigenvalue is found. The histogram curve of a random matrix, unlike this one from real data, can be predicted exactly. For a random matrix there is never an eigenvalue >2.0. The histogram of the empirical stock price data, on the other hand, contains a significant number of eigenvalues >2.0. Some are as big as 5.0. These eigenvalues of necessity must correspond to genuine correlations.

The eigenvalue of a matrix has a corresponding eigenvector – a column matrix of 1000 elements – each element of which is a different weight from each of the 1000 stocks. So we can look at the column vectors that correspond to these deviating, genuinely-correlated eigenvalues and ask: What kinds of stocks entered into each of these eigenvectors? What we found, fortunately, has implications for portfolios. If we restart the graph at 2.0 – removing the distortions of the random values – and look at the 20 eigenvalues >2.0, we see that the stocks that make up most of the weights in the corresponding eigenvectors are almost entirely transportation stocks in the first case, almost entirely paper in the second, almost entirely pharmaceuticals in the third, and so on. In other words, the market *automatically* partitions itself into separate business sectors [66–68]. Thus a physicist who knows nothing about the stock market can mathematically partition the economy into separate business sectors!

The sectors and the quantitative degree to which each constituent firm conforms to the sector can be monitored and updated as a function of time, e.g., every 15 min. Firms that belong to the same business sector can be monitored in a kind of rainbow spectrum. The "good" firms sticking to the business sector are assigned to the "violet" end of the spectrum, and the "bad" firms deviating from the sector are assigned to the "red." When a firm first starts to move to the red end, the spectrum starts to deviate, and this alerts the trader to consider action.

7. Unifying the power laws: A first model

There has recently been progress in developing a theory for some of the power-law regularities discussed above. For example, based on a plausible set of assumptions, we proposed a model that provides an explanation for the empirical power laws of return, volume, and number of trades [12]. In addition, our model explains certain striking empirical regularities that describe the relationship between large fluctuations in prices, trading volume, and the number of trades. In our model, large movements in stock market activity arise from the trades of the large participants. Starting from an empirical characterization of the size distribution of large market participants (mutual funds), we show that their trading behavior when performed in an optimal way, generates the power-laws observed in financial data.

Define p_t as the price of a given stock and the stock price "return" r_t as the change of the logarithm of stock price in a given time interval Δt, $r_t \equiv \ln p_t - \ln p_{t-\Delta t}$. The probability that a return is in absolute value larger than x is found empirically to be [10,46]

$$P(|r_t| > x) \sim x^{-\zeta_r} \quad \text{with } \zeta_r \approx 3. \tag{3}$$

Empirical studies also show that the distribution of trading volume V_t obeys a similar universal power law [50],

$$P(V_t > x) \sim x^{-\zeta_V} \quad \text{with } \zeta_V \approx 1.5, \tag{4}$$

while the number of trades N_t obeys [69]

$$P(N_t > x) \sim x^{-\zeta_N} \quad \text{with } \zeta_N \approx 3.4. \tag{5}$$

We develop a model that demonstrates how trading by large market participants explains the above power laws. We begin by noting that large market participants have large price impacts [70–73]. Accordingly, we perform an empirical analysis of the distribution of the largest market participants—mutual funds. We find, for each year of the period 1961–1999, that for the top 10% of distribution of the mutual funds, the market value of the managed assets S obeys the power law

$$P(S > x) \sim x^{-\zeta_S}, \quad \text{with } \zeta_S = 1.05 \pm 0.08. \tag{6}$$

Exponents of ≈1 have also been found for the cumulative distributions of city size [74] and firm sizes [45,75,76], and the origins of this "Zipf" distribution are becoming better understood [31]. Based on the assumption that managers of large funds trade on their intuitions about the future direction of the market, and that they adjust their speed of trading to avoid moving the market too much, we will see that their trading activity leads to $\zeta_r = 3$ and $\zeta_V = 1.5$,

In order to proceed, we: (a) present empirical evidence for the shape of the price impact; (b) propose an explanation for this shape; and (c) show how the resulting trading behavior generates power laws (3)–(5).

7.1. Empirical evidence for the square root price impact of trades

The price impact Δp of a trade of size V has been established to be increasing and concave [60,77]. We hypothesize that for large volumes V, its functional form is

$$r = \Delta p \simeq kV^{1/2} \tag{7}$$

for some constant k.

A direct statistical test of this hypothesis can be performed by analyzing $E\left[r^2 \mid V\right]$. Performing this regression using r and V calculated over 15 min intervals, we find

$$E\left[r^2 \mid V\right] \sim V. \tag{8}$$

This regression is, however, not definitive evidence for Eq. (7). This regression is performed in fixed Δt, and so is exposed to the effect of fluctuations in the number of trades – i.e. if N denotes the number of trades in Δt, $r^2 \sim N$ and $V \sim N$ so Eq. (8) could be a consequence of this effect.

Since relation (7) implies $P\left(r > x\right) \sim P\left(kV^{1/2} > x\right) = P\left(V > x^2/k^2\right) \sim x^{-2\zeta_V}$, it follows that

$$\zeta_r = 2\zeta_V. \tag{9}$$

Thus, the power law of returns, Eq. (3), follows from the power law of volumes, Eq. (4), and the square root form price impact, Eq. (7).

Recent work by Farmer and Lillo [78] reports an exponent ≈0.3 for the price impact function. Apart from the quality of scaling and the limited data, one possible problem with this estimation is that large trades are usually executed in smaller traunches [79]. Ref. [78] also reports that the volume distribution is not a power-law for the LSE. Further work by Ref. [79] has made it clear that the finding of Ref. [78] of a non-power-law distribution of volume is an artifact arising from incomplete data due to the exclusion of upstairs market trades.

We next develop a framework for explaining Eqs. (4) and (7).

7.2. Explaining the square root price impact of trades, Eq. (7)

We consider the behavior of one stock, whose original price is, say 1. The mutual fund manager who desires to buy V shares offers a price increment Δp, so that the new price will become $1 + \Delta p$. Each seller i of size s_i who is offered a price increase Δp supplies the fund manager with q_i shares. Elementary considerations lead us to hypothesize $q_i \sim s_i \Delta p$. The number of sellers available after the fund manager has waited a time T is proportional to T. So, after a time T, the fund manager can on the average buy a quantity of shares equal to $kT\langle s\rangle \Delta p$ for some proportionality constant k. The search process stops (and the trades are executed simultaneously) when the desired quantity V is reached—i.e., when $kT\langle s\rangle\,\Delta p = V$, so the time needed to find the shares is

$$T = \frac{V}{\langle s\rangle k\Delta p} \sim \frac{V}{\Delta p}. \tag{10}$$

Hence, there is a trade-off between cost Δp and the time to execution T; if the fund manager desires to realize the trade in a short amount of time T, the manager must pay a large price impact $\Delta p \sim V/T$.

Let us consider the manager's decision problem. Managers trade on the assumption that a given stock is mispriced by an amount M, defined as the difference between the fair value of the stock and the traded price [73,80,81]. The manager wants to exploit this mispricing quickly, as he expects that the mispricing will be progressively corrected, i.e. expects that the price will increase at a rate μ. Hence, after a delay of T, the remaining mispricing is only $M-\mu T$. The total profit per share B/V is the realized excess return $M-\mu T$ minus the price concession Δp, which gives

$$B = V\,(M - \mu T - \Delta p)\,. \tag{11}$$

The fund manager's goal is thus to maximize B, the perceived dollar benefit from trading. The optimal price impact Δp maximizes B subject to Eq. (10), $T = aV/\Delta p$, i.e., Δp maximizes $V(M - \mu a V/\Delta p - \Delta p)$, which we will see gives Eq. (7).

The time to execution is $T \sim V/\Delta p \sim V^{1/2}$, and the number of "chunks" in which the block is divided is $N \sim T \sim V^{1/2}$. These effects have been qualitatively documented in [70,71,77]. The last relation gives

$$\zeta_N = 2\zeta_V \tag{12}$$

which in turn predicts $\zeta_N = 3$, a value, approximately consistent with the empirical value of 3.4 [69].

Thus far, we have a theoretical framework for understanding the square root price impact of trades Eq. (7), which together with Eq. (4) explains the cubic law of returns Eq. (3). We now focus on understanding Eq. (4).

7.3. *Explaining the power laws*

Next, we show that returns and volumes are power-law distributed with tail exponents

$$\zeta_r = 3, \qquad \zeta_V = 3/2 \tag{13}$$

provided the following conditions hold: (i) the power law exponent of mutual fund sizes is $\zeta_S = 1$ (Zipf's law); (ii) the price impact follows the square root law (7); (iii) funds trade in typical volumes $V \sim S^{\delta}$ with $\delta > 0$; (iv) funds adjust trading frequency and/or volume so as to pay transactions costs in such a way that defining

$$c(S) \equiv \frac{\text{Annual amount lost by the fund in price impact}}{\text{Value } S \text{ of the assets under management}} \tag{14}$$

then $c(S)$ is independent of S for large S.

The empirical validity of conditions (i) and (ii) was shown above, while condition (iii) is a weak, largely technical, assumption. Condition (iv) means that funds in the upper tail of the distribution pay roughly similar annual price impact costs $c(S)$, and reaches an asymptote for large sizes. We interpret this as an evolutionary "survival constraint". Funds that had a very large $c(S)$ would have small returns and would be eliminated from the market. The average return $r(S)$ of funds of size S is independent of S [82]. Since small and large funds have similarly a low ability to outperform the market, $c(S)$ is also independent of S.

For each block trade $V(S)$, a fund of size S incurs a price impact proportional to $V\Delta p$ which, from condition (ii), is $V^{3/2}$. If $F(S)$ is the fund's annual frequency of trading, then the annual loss in transaction costs is $F(S)\cdot V^{3/2}$, so

$$c(S) = F(S)\cdot [V(S)]^{3/2}/S. \tag{15}$$

Condition (iv) implies that either $V(S)$ or $F(S)$ will adjust in order to satisfy

$$F(S) \sim S \cdot [V(S)]^{-3/2}\,. \tag{16}$$

Condition (i) implies that the probability density function for mutual funds of size S is $\rho(S) = -\partial G/\partial S \sim S^{-2}$. Since condition (iii) states that $V \sim S^{\delta} > x$, and since they trade with frequency given $F(S)$ in Eq. (16),

$$P\,(V > x) \sim \int_{S^{\delta}>x} F(S)\rho(S)\mathrm{d}S \sim \int_{S>x^{1/\delta}} S^{1-3\delta/2} S^{-2}\mathrm{d}S \sim x^{-3/2} \tag{17}$$

which leads to a power-law distribution of volumes with exponent $\zeta_V = 3/2$. Moreover, from Eq. (9), it follows that $\zeta_r = 3$. In addition, the above result does not depend on details of the trading strategy, such as the specific value of δ.

8. Outlook

No one can predict future trends, but approximate inequalities are sometimes predictable. For example, if physicists collaborate with economists, the result is more likely to be useful, and responsible to appreciating the great body of prior work done by economists. Our econophysics work has benefited from collaborations with energetic economists, including Pammolli and Riccaboni of the Lecco Economics Department [83,84], Canning of the Harvard University Economics Department [34,35], Podobnik from Mathematical Finance in Zagreb [85–89] and Salinger of the Boston University Economics Department [39,75,90–93].

Second, we can learn from the work of the masters in science. In this talk, we mentioned Vilfredo Pareto, a "father" of both scaling and universality in economics. Another great master is Omori, who discovered relations among aftershocks to an earthquake. The possibility that the Omori law governing such aftershocks has its analogs in financial "earthquakes" has been proposed by Lillo and Mantegna and developed in a series of papers by Havlin, Yamasaki, and collaborators [94–98].

Lastly, if physicists are humble they will realize that in some cases what we can contribute of most utility in economics is novel ways of thinking about and analysing data, especially since many methods from mathematical statistics are not focussed on handling the strange behavior of nonstationary functions that obey scale invariance, over a limited region of the range of variables. An example might be time series analysis, where Carbone and her collaborators have significantly extended the commonly-used detrended fluctuation analysis (DFA) [99–103] to a new approach called the detrended moving average (DMA) [104–108].

Not only is it possible for the methods and concepts of statistical physics (and nuclear physics [9,65,67]) to influence economic thought, it is also possible that methods and concepts of economics can influence physics thought. In particular, there are notable "unsolved physics problems" such as turbulence, for which recent work on parallels reveals both qualitative similarities [109] and quantitative differences [110,111]. Similarly, problems of economics have stimulated new ideas in mathematics, such as the truncated Lévy walk [87,88,112–115]. Finally, physicists strive to find universality [17,18,116] in economic phenomena [117], and most recent work (but not all work [118]) supports this idea.

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