Seismology-A Statistical Vignette

David Vere-Jones

Journal of the American Statistical Association, Vol. 95, No. 451 (Sep., 2000), 975-978.

lems also have a sequential aspect and provide the opportunity to use adaptive methods that improve over time.

5. CLOSING REMARKS

Popular views of statistics in the geophysical sciences often focus on spatial methods (e.g., Kriging) and time series. Of course, these remain standard methods for data analysis, and their extension to nonstationary and non-Gaussian processes pose new research problems. Recently, Bayesian hierarchical models, coupled with Markov chain Monte Carlo for sampling posteriors, have gained prominence as important modeling tools (Wikle, Milliff, Nychka, and Berliner, unpublished manuscript, 2000). One strength of hierarchical models is the ease with which they incorporate the physical constraints of geophysical processes and their clarity of interpretation. I hope that this vignette balances some conventional views of geostatistics with an emphasis on the emerging areas in atmospheric science.

Due to limited space, I have focused solely on the atmosphere. But a complete understanding of the earth's environment must include chemical and biological processes along with study of the sun. Overlaid on this natural framework is the influence on physical systems by human activities. These areas challenge statisticians with the need to work closely with substantive numerical models and large, complicated observational datasets. I hope that it is clear that no single discipline alone can approach these problems: The easy stuff has been done! Progress in understanding and forecasting the earth's systems requires collaborative effort among teams of scientists, including statisticians.

REFERENCES

Bailey, B. A., Berliner, L. M., Collins, W., and Kiehl, J. T. (1999), "Neural Networks: Cloud Parameterizations," in *Studies in the Atmospheric Sciences*, eds. L. M. Berliner, D. W. Nychka, and T. Hoar, New York: Springer-Verlag.

Berliner, L. M. (1992), "Probability, Statistics, and Chaos," *Statistical Science*, 7, 69–122.

Berliner, L. M., Levine, R., and Shea, D. (1999), "Bayesian Detection of Climate Change," unpublished manuscript.

Berliner, L. M., Nychka, D. W., and Hoar, T. (1999), *Studies in the Atmospheric Sciences*, New York: Springer-Verlag.

Boville, B. A., and Gent, P. (1998), "The NCAR Climate System Model, Version One," *Journal of Climate*, 11, 1115–1130.

Keihl, J. T., Hack, J. J., Bonan, G. B., Boville, B. A., Williamson, D. L., and Rasch, P. J. (1998), "The National Center for Atmospheric Research Community Climate Model: CCM3," *Journal of Climate*, 11, 1131–1149.

Salby, M. L. (1996), *Fundamentals of Atmospheric Physics*, San Diego: Academic Press.

Tebaldi, C., Nychka, D., Brown, B., and Sharman, R. (1999), "Predicting Clear-Air Turbulence," in *Studies in the Atmospheric Sciences*, eds. L. M. Berliner, D. W. Nychka, and T. Hoar, New York: Springer-Verlag.

Seismology—A Statistical Vignette

David Vere-Jones

1. INTRODUCTION

Geophysics, and seismology in particular, has a somewhat uneasy relationship with statistics. On one side, geophysicists have always been passionate collectors, processors, and interpreters of observational data. From Halley in the 17th century to Harold Jeffreys in the 20th century, its leading practitioners have also pioneered important developments in statistics—in graphical and numerical methods, in the treatment of errors, in time series analysis, and in many more specialized topics. On the other side, geophysicists, in common with other physical scientists, tend to cling to a view of the universe as governed by deterministic differential equations. Probability models tend to be relegated to the role of describing observational errors, even where, as in describing the occurrence times of earthquakes, the sources of uncertainty lie considerably deeper. The upshot is that the general level of statistical usage among geophysicists, and among seismologists in particular, is very uneven, from contributions of fundamental importance to disappointing misunderstandings.

The role of seismology within geophysics is greater than its rather special subject matter might suggest. This is chiefly because for many years, measurement of the reflections and refractions of earthquake waves passing through the earth has provided an important tool for probing the earth's inner structure. At the same time, the subject is kept in the public eye through its applications to engineering, building codes, insurance, and earthquake prediction. In all of these applications, and at the heart of the subject itself, statistical problems abound, few of them easy and some challenging the limits of current statistical methodology. In this vignette I attempt to indicate the nature of these problems, first by tracing a brief history of seismology, and then by selecting a few special issues of current interest. Bolt (1988) and Bullen (1963) have provided useful general introductions to the subject.

2. A BRIEF HISTORY OF SEISMOLOGY, WITH A STATISTICAL BIAS

2.1 First Stages: 1890–1920

Seismology has little statistical history before the development of the Milne–Shaw seismograph in the 1890s. Somewhat earlier the theory of wave propagation in an elastic medium had been worked out, a qualitative inten-

---

David Vere-Jones is Emeritus Professor, School of Mathematical and Computing Sciences, Victoria University, Wellington, New Zealand.

Journal of the American Statistical Association
September 2000, Vol. 95, No. 451, Vignettes

sity scale for assessing ground motion had been developed, and there had been attempts to compile lists of large historical earthquakes. But the Milne–Shaw instrument was the first that was compact and accurate enough to allow objective measurements of earthquake wave motion in many different places to be made and compared. Early instruments were deployed not only in Europe and the United States, but also in Japan, New Zealand, and China.

With the first reliable instruments came the first reliable data and the beginnings of seismology as a quantitative scientific discipline. As early as 1894, the Japanese seismologist Omori, studying the aftershocks of large Japanese earthquakes, formulated the first empirical law of seismology, *Omori's law*: the frequency $\lambda(t)$ of aftershocks at time t after the main shock decays hyperbolically. This is most commonly quoted in the generalized form

$$\lambda(t) = \frac{A}{c}\left(1 + \frac{t}{c}\right)^{-(1+\delta)}.$$

Remarkably, to this day, Omori's law remains without any clear physical explanation. It can be modeled to a first approximation as a nonhomogeneous Poisson process (Jeffreys 1938), and it is used in Ogata's epidemic-type aftershock sequence (ETAS) model (Ogata 1988), where every event is supposed to trigger its own aftershocks ("offspring"), but the models remain at a descriptive level.

The next major stimulus was the 1906 San Francisco earthquake. Challenging U.S. technical "know-how" at one of its major centers, the earthquake drew forth a massive technical report (Lawson 1908) that documented the extent and character of damage and of displacements along and off the San Andreas fault, and thereby laid the foundations for the field of engineering seismology. In a sequel to this report, H. F. Reid (1911) set out his *elastic rebound theory* of earthquakes: On either side of a major fault, large-scale forces operate to cause relative motion between the two sides of the fault. Friction opposes the motion. As time passes, the rock material deforms elastically and strain (deformation), and hence stress (elastic force), accumulate, until ultimately the strength of the fault is exceeded and the two sides slip in an earthquake; then the process starts again. Detailed measurements from surveys before and after the 1906 earthquake strongly supported this hypothesis, which both explained the origin of the earthquake waves and gave some basis for regarding earthquakes as a recurrent process.

In broad terms, Reid's hypothesis has dominated thinking about earthquake mechanisms ever since its formulation. It lies behind recent stochastic models for earthquake occurrence, such as renewal or semi-Markov processes with log-normal interevent times, or the stress-release model, in which the conditional intensity at time t has the form

$$\lambda(t) = \exp[a + bX(t)],$$

where $X(t) = X(0) + \rho(t - \gamma \sum_{t_i<t} X_i)$ is a measure of the current stress level in the region, the X_i being the stresses released in previous events. Once again, however, the models remain at a broadly descriptive level.

2.2 The Classical Period: 1920–1950

The decades following the 1906 San Francisco earthquake were marked by steady improvements in instrumentation and data collection. Networks of stations were established, and information began to be collected at both global and local levels. The principal theme was not the study of the earthquakes themselves, however, but rather the information that they provided about the earth's interior. As early as 1909, Mohorovicic had observed waves apparently reflected from an internal boundary some kilometers below the earth's surface, and evidence for other boundaries, including that of a central core, accumulated.

The disentanglement of such data is a classical inversion problem, the basic unknown being the velocity structure inside the earth. Many seismologists contributed to these issues, but the most profound contributions were those of Harold Jeffreys, later assisted by K. E. Bullen. The notable feature of Jeffreys's work was its careful attention to statistical procedures. Like that of Laplace and Gauss before him, Jeffreys's fundamental work on probability and statistical inference (Jeffreys 1939) was underlain by the better part of a decade of experience in the reduction of physical data, earthquake travel-time data and the establishment of improved procedures for epicenter and hypocenter (three-dimensional) location. The work culminated in 1940 with the publication of the Jeffreys–Bullen global travel-time tables (Jeffreys and Bullen 1940), which are still used for calculating travel times based on the assumption of a spherically symmetric globe.

Another important step taken during this period was Richter's (1935) development of an earthquake magnitude scale. Based on the logarithm of the maximum amplitude recorded on a standard instrument, and adjusted to a standard distance from the source, this was the first objective measure of the size of an earthquake. Despite its limitations, such a measure remains an almost indispensible tool for the quantitative analysis of earthquake catalog data.

Hard on the heels of the magnitude scale came the second empirical law of seismology, the *Gutenberg–Richter frequency-magnitude law*. In statistical terms, this law asserts that magnitudes follow an exponential distribution. If the magnitudes are related back to physical variables such as the seismic energy release, then this translates to a power law (Pareto's law) distribution for the physical variable. In particular, it suggests that the tails of the energy distribution are of the form

$$\mathrm{pr}(\text{energy} > E) \propto E^{-\alpha},$$

where the exponent α is in the range .4–.8.

This law also remains without any universally accepted explanation, although the problem here is less an absence than a proliferation of models. It puts earthquakes squarely into the realm of phase-change–like phenomena, associated with features such as power law distributions of size, long-range spatial and temporal correlations, and self-similarity. But here there are many possible models, and the Gutenberg–Richter (G-R) law by itself is not enough to distinguish between them. Nevertheless, it and Omori's law provide constraints that any successful model of the earthquake fracture process must satisfy.

2.3 Time Series Analysis and Explosions: 1945–1970

Time series and geophysics have grown up together, each contributing to the development of the other, and seismology has been an integral part of this process. Early time-series work in seismology related to the largely fruitless study of hidden periodicities in earthquake occurrence. In the period following the Second World War, however, a number of practical problems pushed time-series methods into the center of seismological research. The most important of these (not in the least because it led to substantial increases in funding for seismology) was the problem of detecting underground nuclear explosions and distinguishing them from earthquakes. The analysis of data from seismic arrays (i.e., instruments set up in a grid or other structured pattern) required the solution of further problems. The same period saw the growing use of explosion seismology (recording of waves from deliberate explosions) to investigate subsurface structures for oil exploration and other purposes, and of spectral methods to analyze the response of buildings and other structures to earthquake waves. As seismic networks became more highly automated, questions arose concerning the automatic triggering of unmanned equipment and the effective analysis and storage of data from such equipment. All of these issues required the solution of difficult, often highly technical problems in time series analysis and engaged the attention of leading experts in both fields. New ideas arose, such as maximum entropy methods, and the links between the disciplines remain very close.

2.4 Plate Tectonics and Earthquake Prediction: 1970–Present

Plate tectonics is one of the scientific success stories of the second half of the twentieth century. For seismology, it provided a unifying principle that helped explain many incompletely resolved issues. It gave meaning to the highly irregular distribution of seismically active zones around the world, and indicated the nature of the "large-scale forces" required by Reid's elastic rebound theory—plate motions, impelled by convection processes in the earth's mantle. Collision and subduction zones, rigid plates and fractured plate-boundaries, mid-ocean ridges, and heat flow and gravity anomalies were concepts illuminated and coordinated by plate tectonics.

This somewhat euphoric period also saw the first steps in what was to prove a salutory reminder that the earth does not yield its secrets cheaply. In the lull before the Chinese cultural revolution, intriguing rumors of eccentric animal behavior and anomolous physical measurements before large earthquakes emanated from behind the "bamboo curtain." These culminated in 1976 with the claimed prediction of the Haicheng earthquake, leading to the evacuation of residents from their homes and the consequent saving of many lives. Observers from the international seismological community visited and confirmed much of the story. American and Japanese scientists, conscious of the superiority of their technical equipment, were spurred to emulate the Chinese, again assisted by offerings of additional funds. Only 2 years afterward, however, the Chinese program suffered a severe reversal with the devastating 1978 Tangshan earthquake. No formal predictions were claimed, and massive losses of both life and property were incurred. Such has been the progress of earthquake prediction ever since; each claimed success has been matched by an embarrassing failure to predict or a false alarm. The unpredicted earthquakes in Northridge, California and Kobe, Japan, each in the heart of earthquake research territory, did little to help matters. Funding started to dry up, the credibility of scientists working on prediction was threatened, and serious doubts were entertained as to whether earthquake prediction was a feasible or even a desirable accomplishment.

I am more optimistic over these matters than the last paragraph might suggest (Vere-Jones 1995). The viewpoint is slowly gaining acceptance that predictions must be couched in terms of probabilities of occurrence. Many embarrassments might have been avoided had this viewpoint prevailed sooner. Currently, there seem to me two main stumbling blocks. The first is in the physics, in the lack of an adequate theory of earthquake genesis and growth. The second is the lack of statistical models for the highly clustered, self-similar types of data from earthquake patterns. In both areas there is considerable room for improvement and some indication that despite current pessimism, the problems are starting to yield. Features such as local activation, foreshocks and precursory swarms, accelerated moment release, and precursory quiescence do provide some degree of enhancement of background probabilities, and suggest that the accumulation of stress before a large event may be detectable. However, the factors are not yet large enough, and the models are not well enough established, for them to be useful in direct practical applications. Improvements in data quality and the range of characteristics studied can only lead to improvements in this situation.

In the meantime, seismology offers statisticians the opportunity to collaborate in an extremely diverse range of problems. Let me conclude by quoting a few examples of recent or current work which happen to have caught my interest (but are not claimed to be representative).

3. SOME RECENT EXAMPLES

Dating of Events Along the New Zealand Alpine Fault. Although this fault marks a major plate boundary, it has been a seismically quite zone ever since Europeans arrived about 2 centuries ago. A central issue was to determine whether large earthquakes had occurred along the fault, and if so, when. A recent workshop brought together scientists who had been tackling this issue from different points of view: carbon dating from peat residues, tree ring data, data from lichens growing on the underside of fallen rocks. As the workshop progressed, the uncertainties in one technique were resolved by information provided by another. By the time it finished, a clear answer had emerged—the last major earthquake had occurred in 1717. Before then, two or three further dates were established, with less certainty, at intervals of from 100 to 300 years.

A Time Series Problem: Identification of Preseismic and Coseismic Changes in Water Well Levels. Water level changes have long been touted as an earthquake precursor. Kitagawa and Matsumoto (1996) finally married data of sufficient quality to noise-reduction techniques (involving nonlinear filtering) of sufficient sensitivity to isolate coseismic and some small preseismic signals.

Inversion Problems: Mapping of Slip at Depth and Gravity Anomalies. The 1989 Loma Prieta earthquake was widely felt and caused moderate damage; it did not, however, break the surface. Using a combination of statistical and geophysical arguments, Arnodottir, Segall, and Matthews (1992) were able to reconstruct the area on the fault plane that slipped, from measurements taken on the surface across and alongside the fault. In somewhat related work, Bayesian smoothing methods developed by Akaike and coworkers have been used to tackle a wide range of geophysical inversion problems; one example is their use by Murata (1992) to map Bouguer density anomalies.

Forecasting of Large Aftershocks. Matsu'ura (1986) studied the forecasting of large aftershocks using Ogata's techniques, based on the ETAS model, for detecting precursory relative quiescence. This was one of the few techniques to perform credibly in the Kobe earthquake, in which it gave real-time warnings of major aftershocks.

Fundamental Theory: Mode-Switching in Complex Earthquake Models. A key theoretical issue is the tussle between rebound-type arguments, suggesting "characteristic earthquakes" at regular intervals, and the G–R law, suggesting extreme irregularity. In a series of recent articles (e.g., Dahmen, Ertas, and Ben-Zion 1998), Ben-Zion and colleagues in the United States have demonstrated the existence of complex systems, imitating features of tectonically driven fault structures, and capable of existing in two possible modes. The first mode produces more or less stationary sequences of events following a standard G–R law. The second mode exhibits near periodic behavior of the elastic-rebound type, with regular occurrence of large "characteristic" events outside the G–R range. The process can flip from one mode to the other at apparently random instants of time. Does the geological evidence support the existence of such behavior in real fault systems?

Statistical Seismology and an S-PLUS–Based Software Environment. Recent work of our own group has focussed on reviewing the applicability of catalog-based prediction methods to New Zealand data. Working jointly with a group headed by Professor Ma Li from the Chinese Seismological Bureau, we have developed an S-PLUS–based software environment (SSLIB; see Harte 1999) for subsetting and displaying catalog data and for fitting, simulating, predicting, and evaluating a range of conditional-intensity models.

REFERENCES

Arnadottir, T., Segall, P., and Matthews, M. (1992), "Resolving the Discrepancy Between Geodetic and Seismic Fault Models for the 1989 Loma Prieta, California, Earthquake," *Bulletin of the Seismologic Society of America*, 82, 2248–2255.

Bolt, B. A. (1988), *Earthquakes* (2nd ed.), San Francisco: W. H. Freeman.

Bullen, K. E. (1963), *An Introduction to the Theory of Seismology* (3rd ed.), Cambridge, U.K.: Cambridge University Press.

Dahmen, K., Ertas, D., and Ben-Zion, Y. (1998), "Gutenberg Richter and Characteristic Earthquake Behavior in Simple Mean-Field Models of Heterogeneous Faults," *Physical Reviews, E*, 58, 1494–1501.

Harte, D. (1999), "Documentation for the Statistical Seismology Library," Research Report 98-10, School of Mathematics and Computer Science, Victoria University, Wellington.

Jeffreys, H. (1938), "Aftershocks and Periodicity in Earthquakes," *Gerlands Beitrage zur Geophysik*, 53, 111–139.

——— (1939), *The Theory of Probability*, Oxford, U.K.: Oxford University Press.

Jeffreys, H., and Bullen, K. E. (1940), *Seismological Tables*, British Association, Gray-Milne Trust.

Kitagawa, G., and Matsumoto, N. (1996), "Detection of Coseismic Changes of Underground Water Level," *Journal of American Statistical Association*, 91, 521–528.

Lawson, A. C. (Chairman) (1908), *The California Earthquake of April 18th 1906. Report of the State Earthquake Investigation Commission*, Vols. 1 and 2, Washington, Carnegie Institute of Washington.

Matsu'ura, R. S. (1986), "Precursory Quiescence and Recovery of Aftershock Activities Before Some Large Aftershocks," *Bulletin of the Earthquake Research Institute, University of Tokyo*, 61, 1–65.

Murata, Y. (1992), "Estimation of Optimum Surface Density Distribution Only From Gravitational Data: An Objective Bayesian Approach," *J. Geophys. Res.*, 12097–12109.

Ogata, Y. (1988), "Statistical models for earthquake occurrences and residual analysis for point processes." *Journal of American Statistical Association*, 83, 9–27.

Omori, F. (1894), "On After-Shocks of Earthquakes," *Journal of the College of Sciences*, Tokyo Imperial University, 7, 111–200.

Reid, H. F. (1911), "The Elastic Rebound Theory of Earthquakes," *Bulletin of the Department of Geology, University of California*, 6, 413–444.

Richter, F. (1935), "An Instrumental Earthquake Scale," *Bulletin of the Seismologic Society of America*, 25, 1–32.

Vere-Jones, D. (1995), "Forecasting Earthquakes and Earthquake Risk," *International Journal of Forecasting*, 11, 503–538.

Vere-Jones, D., and Smith, E. G. C. (1981), "Statistics in Seismology," *Communications in Statistics, Part A—Theory and Methods*, A10, 1559–1585.